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Shareholders' Wealth-Maximizing Operating Decisions and Risk Management Practices in a Mixed Contracts Economy

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This paper establishes a critically important positive role for operations management practices and financial hedging. We show that operations management decisions and financial hedging are intertwined, and we advance a framework that can identify their combined effects on investors' wealth. We show that: (a) firms (publicly traded corporations) will optimally hold adequate riskless working capital (e.g., cash) to minimize the cost of obtaining non-financial inputs, and the magnitude of this cash holding depends on operating details, and (b) operations management and financial hedging can lower firms' cash requirements, and boost productivity, defined as the wealth created in the firm per dollar of invested capital. Productivity-enhancing practices—by "freeing up" some of the firm's cash—can maximize the investors' wealth. We show that these results obtain because firms' contracts with many of the providers of non-financial inputs are not traded, and because investors can invest not just in public corporations but also in businesses "outside the markets" (e.g., proprietorships, partnerships, and private equity).

Key words: risk management; financial hedging; operational hedging; operations management; working capital; cash; mixed contracts; factor contracts; shareholder wealth; productivity of capital; lean production; non-tradability premium
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1. Introduction and Overview

In this research we link operations management practices and financial hedging to the shareholders' wealth, in a more general setting than in the prior research. We develop an analytical model in which the firm's cash flows are determined, endogenously, by the firm's optimal production and risk-management decisions, and show how these decisions affect the shareholders' wealth.

In our framework, there are two types of economic institutions—firms (public corporations) and private businesses (e.g., proprietorships, partnerships, and private equity) that exist "outside the markets."[1] Our analysis focuses on a public firm ("the firm") and on the firm's owners ("the shareholders"). The firm is run by a manager. The firm and the firm's manager are indistinguishable; their role is to engage in and execute contracts that maximize the wealth of the shareholders. We make no assumptions about the manager's/firm's utility function. We assume that the manager abides by his fiduciary duty to maximize the shareholders' wealth; thus, we exclude agency issues from the analysis.

There are two types of economic agents—investors with financial capital, and "factors" who own non-financial inputs. Investors can supply funds to firms, in which case, they are "shareholders" holding market-traded securities (stock). They can also take ownership positions in businesses outside the markets, in which case, they will hold non-traded private ownership contracts.

The factors provide their inputs to the firm today in return for a promised payment next period that is set exogenously. The factors' contracts (e.g., contracts for the supply of labor services, land, raw materials, processed goods, and supplies) are not traded in the markets.[2]

In this setting, which we describe as a "mixed contracts economy" (traded securities and non-traded contracts), we focus on the wealth impact, for shareholders, of operations and risk-management decisions in the firm. The only source of risk for the firm's cash flows in our model comes from demand uncertainty. We assume there are no market imperfections (frictions), no arbitrage opportunities in the securities markets, and that stock prices satisfy the Arbitrage Pricing Theory (APT). Since factor contracts and ownership contracts in private business are not traded, these contracts need not satisfy the APT.

The essence of our economic argument is as follows. We argue that the firm will benefit its shareholders by holding, depending on its operating details, adequate riskless working capital (cash); by holding cash the firm can reduce the cost of obtaining the non-financial inputs from the factors. Operations management decisions, and financial hedging, by changing the probability distribution of the firm's cash flows, can lower the firm's cash holdings, and "free up" cash that can be returned to its shareholders. The shareholders can then further augment their wealth by using this cash to invest in business opportunities outside the markets.

Our results are not driven by the usual market frictions (e.g., agency/bankruptcy costs, taxes, informational asymmetries). They stem entirely from allowing the existence of operating risk (defined as negative operating earnings), mixed contracts (securities and factor contracts) and from recognizing that investors have investment opportunities outside the publicly traded corporation (in other organizational forms).[3]

We show how operations management and risk-management decisions affect: (a) the firm's riskless working capital (e.g., cash) requirements, (b) the (covariance) risk of the cash flows, (c) the value of the firm, (d) the firm's productivity of capital, which we define as the wealth (NPV) created per dollar of invested capital (which includes the working capital required), and (e) the shareholders' total wealth, which includes the wealth created inside the firm plus the wealth they can derive from investment opportunities in private business ventures.

The rest of this paper contains six sections. In section 2, we review the related operations management research and provide contextual positioning of this paper's contributions. In section 3, we describe our assumptions about the firm's operating income. We then explain why and how the firm can, by holding adequate cash, lower the cost of obtaining the non-financial inputs from the factors. We then identify the covariance risk of the firm's cash flows, and firm value, when the firm holds cash. In section 4, we examine the implications of a production plan on the investors' total wealth (their combined wealth in and outside the firm). We show that to identify the firm's optimal production plan the operations manager must examine both the wealth that the plan creates inside the firm and the firm's productivity of capital. We provide numerical examples to illustrate our economic arguments. In section 5, we show that financial hedging practices can augment the shareholders' total wealth. Specifically, we show how hedging decisions can change the firm's riskless working capital requirements, alter the firm's optimal production plan, and increase the shareholders' total wealth. In section 6, we discuss how the extant results in the literature obtain as special cases of our framework. Section 7 contains a summary of our results and closing comments.

2. Literature Review

Although a clean taxonomy is difficult because of overlapping model features, the related operations management literature can be thought of as having evolved along three research strands.

The first strand addresses operational hedging—broadly defined as mitigating the impact of risk through operational decisions. This research stream includes papers that address currency risk management in an international production network (Gutierrez and Kouvelis 1995, Huchzermeier and Cohen 1996), research that focuses on demand risk (Lee and Tang 1997, 1998, Van Mieghem and Dada 1999), and work that centers on both demand and currency risk management (Kazaz et al. 2005, Kouvelis and Gutierrez 1997). For a survey of operational hedging studies related to capacity decisions, see Van Mieghem (2003). All of these studies assume that the manager's goal is to maximize the firm's expected cash flow.

A second strand of the literature (Agrawal and Seshadri 2000, Bouakiz and Sobel 1992, Caldentey and Haugh 2006, Ding et al. 2007, Eeckhoudt et al. 1995, Gaur and Seshadri 2005, Sethi 1997) identifies optimal operating and hedging policies by making specific assumptions about the manager's utility function.

The third line of inquiry (Anvari 1987, Berling and Rosling 2005, Birge 2000, Birge and Zhang 1999, Lederer and Singhal 1988, Singhal 1988) explores how a firm's operating decisions affect its financial risk, and hence its market value. A subset of papers in this literature examines operations management and other financial decisions by invoking market frictions. For example, MacMinn (1987) and Taksar (2000) study corporate insurance in a model that includes bankruptcy and agency costs. Xu and Birge (2004) integrate production and financing decisions by balancing the tax benefits of debt against financial distress costs. Taksar (2000) studies an insurance company's cash surplus, and its implications for a firm's dividend policy, and Buzacott and Zhang (2004) integrate the firm's production and financing decisions in a model where the firm has limited capital and is restricted to bank financing.

Our research differs from the first research stream in two ways: (1) it combines both operational and financial hedging, and (2) the optimization analysis takes place within the (no-arbitrage) value-maximization framework. It differs from the second set of studies in that it does not rely on specific assumptions about the manager's utility function. Finally, in contrast to studies in the third stream, we do not rely on the usual market frictions (e.g., bankruptcy costs, taxation and agency issues).

3. Firm Value, Riskless Capital, and Shareholders' Total Wealth in a Mixed Contracts Economy

3.1. Model of Operating Income

We assume that the firm produces n products for which demand is stochastic, and we denote by $\mathbf{q} = (q_1, \ldots, q_n)$ and $\mathbf{d} = (\xi_1, \ldots, \xi_n)$ the vectors of production quantities and random demands, respectively. The only source of risk in our model comes from demand uncertainty. We define the firm's operating income as

$$\tilde{X}(\mathbf{q}, \mathbf{d}) = \tilde{R}(\mathbf{q}, \mathbf{d}) - C(\mathbf{q}, \mathbf{d}), \tag{1}$$

where $\tilde{R}(\mathbf{q}, \mathbf{d})$ is the revenue earned by supplying $\mathbf{d}$ when the production plan is $\mathbf{q}$, and $C(\mathbf{q}, \mathbf{d})$ is the cost of producing $\mathbf{q}$ when the demand outcome is $\mathbf{d}$. The operating income $\tilde{X}$ is, in general, a function of the production plan $\mathbf{q}$, and it is a random variable that depends on the random demand. We write the operating income as $\tilde{X}(\mathbf{q}, \mathbf{d})$ to emphasize its dependence on both the production plan $\mathbf{q}$, and demand $\mathbf{d}$. That is, $\tilde{X}(\mathbf{q}, \mathbf{d})$ is the operating income obtained if we produce $\mathbf{q}$ and the demand realization is $\mathbf{d}$. However, to simplify notation, and where there is no ambiguity, we write it as $\tilde{X}(\mathbf{q})$.

We denote the firm's set of production possibilities as $\mathcal{Q}$, and assume that $\mathcal{Q}$ is a convex set. We further assume, without loss of generality, that any production plan $\mathbf{q}$ that cannot be profitably produced when it is demanded is not in $\mathcal{Q}$, that is, $\mathbf{q} \in \mathcal{Q}$ only if $\tilde{R}(\mathbf{q}, \mathbf{q}) \geq C(\mathbf{q}, \mathbf{q})$. We call a production plan $\mathbf{q}$ feasible if $\mathbf{q} \in \mathcal{Q}$; similarly, we call a demand outcome, $\mathbf{d}_0$, feasible if $\mathbf{d}_0 \in \mathcal{Q}$.

We assume that the initial investment required at time t, I, is constant for all operating plans, and that αI, $0 \leq \alpha \leq 1$, of the initial investment is recovered at $t+1$.

The selection of a myopic (i.e., single–period) production plan $\mathbf{q}$ can be thought of as the selection of a project among a set of mutually exclusive alternatives, all with a duration of a single period. To identify the firm's optimal production plan, it is important to evaluate the marginal effects of changes in q_i on the firm's operating income, on its cash flow risk, on the firm's value, and ultimately on the shareholders' wealth. To this end, we make three assumptions about $\tilde{X}(\mathbf{q}, \mathbf{d})$.

Assumption A1. *Operating Income is Monotonic in Demand.* We assume that operating income $\tilde{X}(\mathbf{q}, \mathbf{d})$ is non-decreasing in ξ_i for any production plan $\mathbf{q}$.

Assumption A2. *Operating Income has Positive Mixed Partial Derivatives.* We assume that the partial derivative $\frac{\partial \tilde{X}(\mathbf{q}, \mathbf{d})}{\partial q_i}$ is non-decreasing in $\mathbf{d}$ for all products i.

This assumption requires that $\frac{\partial^2 \tilde{X}(\mathbf{q}, \mathbf{d})}{\partial \xi_j \partial q_i} \geq 0$ for any products i and j.[4]

Assumptions (A1) and (A2) together imply that both the operating income and the marginal change in operating income resulting from increasing production of an item i will never decrease when the demand for any item j, possibly $j = i$, increases.

Assumption A3. *Excess Production is Costly.* We assume that for any two production plans $\mathbf{q}_1$ and $\mathbf{q}_2$ and a given demand outcome $\mathbf{d}_0$ with $\mathbf{d}_0 \leq \mathbf{q}_1 \leq \mathbf{q}_2$, the resulting firm's operating incomes are ordered as $\tilde{X}(\mathbf{q}_2, \mathbf{d}_0) \leq \tilde{X}(\mathbf{q}_1, \mathbf{d}_0)$.

Assumptions (A1)–(A3) are parsimonious; they allow for the production plan, $\mathbf{q}$, to be determined with varying degrees of demand information. They thus admit Make-To-Stock (MTS) production models (i.e., when $\mathbf{q}$ must be selected before observing demand $\mathbf{d}$), and Make-To-Order (MTO) models (i.e., when $\mathbf{q}$ can be selected after observing $\mathbf{d}$) as special cases. In an MTS model, we can assume that the cost of production is determined before observing demand; hence, it is constant across all demand outcomes. Assumptions (A1)–(A3) also allow the modeling of systems with product substitution and production systems with non-linear costs (e.g., economies or diseconomies of scale and some cost interactions across products). Finally, we point out that since $\tilde{X}(\mathbf{q}, \mathbf{d})$ is a cash-flow, it excludes any non-monetary costs imputed to inventory shortages (such as penalty costs derived from customer "ill-will").

We now provide two simple examples of MTS and MTO production systems to illustrate the above assumptions.

Example 1 (MTS System). In this production system, the production plan $\mathbf{q}$ is specified before demand $\mathbf{d}$ is revealed. Thus demand is satisfied up to the quantity produced and excess inventories (unsold output) are disposed of at a salvage price of v_i per unit. The selling price of an item is denoted as p_i, and the unit cost of producing a unit of product i is denoted as c_i. The resulting firm's operating income, $\tilde{X}$, is thus

$$\tilde{X}(\mathbf{q}, \mathbf{d}) = \sum_{i=1}^{n} \left(p_i \min\{\xi_i, q_i\} + v_i (q_i - \xi_i)^+ - c_i q_i \right). \tag{2}$$

Since the production plan is fixed before observing demand (i.e., the production plan $\mathbf{q}$ is constant in $\mathbf{d}$), the expected cash flow is obtained as $\mathrm{E}\,\tilde{X}(\mathbf{q}, \mathbf{d})$. □

In this example, Assumptions (A1) and (A2) are satisfied if $p_i > v_i$, and Assumption (A3) is satisfied if $v_i \leq c_i$ for all products i and for all feasible demand vectors $\mathbf{d} \in \mathcal{D}$. Further, the set $\mathcal{D}$ is non-empty if $c_i < p_i$ for some i.

Example 2 (MTO System). In this case, we assume demand is revealed before any item is produced. Once the demand for product i is revealed, it is satisfied up to q_i^h. The value of the limit q_i^h may be determined by production capacity or supply constraints. The production plan for product i is $q_i = \min(\xi_i, q_i^h)$. In this case, the production plan is a function of $\mathbf{d}$, and we write it as $\mathbf{q}(\mathbf{d}) = \mathbf{d} \wedge \mathbf{q}^h$, where $\mathbf{q}^h$ is the vector whose ith element is q_i^h. Operating income, $\widetilde{X}$, is given as

$$\widetilde{X}(\mathbf{q}(\mathbf{d}), \mathbf{d}) = \sum_{i=1}^{n} \{(p_i - c_i)(\xi_i \wedge q_i^h)\}. \qquad (3)$$

As in the MTS system, p_i and c_i denote the unit selling price and unit production cost of product i, respectively. □

In the MTO example above, Assumptions (A1)–(A3) are satisfied if $0 \leq c_i \leq p_i$ for all products i and for all feasible demand vectors $\mathbf{d} \in \mathcal{D} \neq \emptyset$.

3.2. A Rationale for Riskless Working Capital (Corporate Cash)

As a starting point for our analysis, we assume that if the firm has no operating risk (which we define as a positive probability of negative operating earnings, revenues less cost of non-financial inputs) it would obtain, at $t = 0$, the necessary non-financial inputs from the "factors"—the owners of these inputs—at a price f (this price is exogenous to our analysis) payable at $t = 1$. In this case, the factors are guaranteed their opportunity costs.

An inevitable question, therefore, is how will the firm optimally provide the factors their opportunity costs if it faces operating risk? The answer, as we show next, depends on the firm's operating risk, and on whether the factor contract is assumed to be a security or a non-traded contract.

At this point it is useful to examine separately the implications of the non-tradability of the factor contract for the case where the firm has no operating risk (Case 1) and the case where the firm has operating risk (Case 2).

Case 1: The Firm has no Operating Risk

With this assumption, the factors are guaranteed to recover their promised payoffs at $t = 1$. Now for explanatory purposes, consider the following two possibilities:

a. *The factor contract is a market-traded security.* In this case, the factors would demand a price $f^T < f$ payable at $t = 1$. The $t = 0$ value of this security is simply the present value of f^T discounted at the risk-free interest rate.
b. *The factor contract is not traded.* In this case, the factors will demand a price f payable at $t = 1$. The difference $\delta_0 = f - f^T > 0$ is a "non-tradability premium" that the factors will demand for holding a non-traded riskless contract. The $t = 0$ value (present value of cost) for the firm of the factor contract is the present value of f discounted at the risk-free interest rate.

In our analysis above, even when there is no risk, factor contracts will command a positive non-tradability premium; given two identical cash flows, an investor will prefer the one that can be capitalized and market-traded. If the factor is to be indifferent between a security paying f^T and a non-traded contract paying f, we must have $\delta_0 = f - f^T > 0$. Hence, even in the absence of operating risk, the firm cannot avoid paying the premium δ_0.[5]

Case 2: The Firm has Operating Risk

In this case, the factors may not recover their promised payments fully. To adjust for this risk, the factors will demand a fixed price f_R payable at $t = 1$ that is *de facto* a state-contingent payoff. Specifically, the factors will receive $f_R(\omega)$ with $f_R(\omega) = f_R$ in those states of the world, ω, where the firm's operating income is non-negative and $f_R(\omega) < f_R$ in states where the firm has negative operating income. Now revisit the two possibilities discussed earlier:

a. *The factor contract is a market-traded security.* In this case, the factors will demand a price f_R that has a risk-neutral expectation, $E^{\mathbb{Q}} f_R(\omega) = f^T$.
b. *The factor contract is not traded.* In this case, the factors will demand a fixed price $f_R^N > f$ where $\delta_R = f_R^N - f > 0$ is a non-tradability premium.[6] Nevertheless, as before, they obtain *de facto* a state-contingent payoff. Specifically, the factors will receive $f_R^N(\omega)$ with $f_R^N(\omega) = f_R^N$ in those states of the world, ω, where the firm's operating income is non-negative and $f_R^N(\omega) < f_R^N$ in states where the firm has negative operating income. The factors will set f_R^N such that their risk-neutral expectation, $E^{\mathbb{Q}} f_R^N(\omega) > f$.[7]

Our discussion above identifies two components of the non-tradability premium, a risk-free component δ_0, and a risk-sensitive component δ_R, and we have provided empirical evidence pointing at the existence of both. Grossman (1995) imputes non-tradability premiums to the *incomplete equitization of risks*, which he defines as the inability of economic agents to trade some of their claims to future cash flows. Although the existence of positive non-tradability premiums has been rationalized, there is no known theory for endogenizing its value. However, if the firm holds enough cash to compensate its worst operating shortfall, it can eliminate the risk to the factors and they will accept f; in this case the firm can avoid paying δ_R.

Since we do not have any theory to quantify the magnitude of the non-tradability premium, δ_R, as a function of the risk and the magnitude of potential shortfalls, we avoid the issue altogether by assuming enough cash is held to compensate its worst operational shortfall and completely eliminate the risk to the factors. By self-insuring, this payment of $\delta_R > 0$ to the factors is eliminated. Note that partial coverage (e.g., 90% instead of 100%) of the shortfall will raise the cost of the non-financial inputs as there is still some residual risk for the factors. Note also that, although the payment of δ_0 is unavoidable, we do not need to determine δ_0 explicitly as it is already included in the negotiated payment f.

In our model the firm "self-insures" against operating risk by holding cash today, in an amount $\mathscr{L}$ equal to the present value of the maximum potential operating shortfall (not the expected shortfall). Thus, assuming the cash is invested at the risk-free interest rate, r_f, the cash required by a production plan $\mathbf{q}$ is given by

$$\mathscr{L}(\mathbf{q}) = \frac{1}{1+r_f} \max\left(-\tilde{X}(\mathbf{q},\mathbf{d}) - \alpha I\right)^+. \tag{4}$$

With the firm holding $\mathscr{L}$ in cash (that is, earning the riskless interest rate r_f), the NPV of this investment is zero (since $\mathscr{L}$ plus the interest is recovered in all states of the world next period). Thus, cash is a mechanism for protecting the factors (*ex ante*).[8] Note that the cash rationale here is endogenous.[9]

It follows from (4) that the amount of riskless working capital required by the firm, expressed as a function of the production plan $\mathbf{q}$ and of the minimum possible demand $\mathbf{d}_m$, is given by

$$\mathscr{L}(\mathbf{q}) = \frac{1}{1+r_f}(-\tilde{X}(\mathbf{q},\mathbf{d}_m) - \alpha I)^+. \tag{5}$$

With cash in the firm, the cash flow to the shareholders at $t+1$, $\tilde{F}$ is defined as

$$\tilde{F}(\mathbf{q}) = \tilde{X}(\mathbf{q}) + \alpha I + (1+r_f)\mathscr{L}(\mathbf{q}). \tag{6}$$

3.3. Characterization of $\mathscr{L}(\mathbf{q})$

To understand how different production decisions affect the wealth created in the firm and the selection of the shareholders' wealth-maximizing production plan, it is important to analyze the function $\mathscr{L}(\mathbf{q})$. In this subsection, we establish properties for $\mathscr{L}(\mathbf{q})$ and then, in section 4, we analyze the implications for shareholders' wealth creation.

We define the *cash-free* production set $\mathscr{Q}_0$, $\mathscr{Q}_0 \subseteq \mathscr{Q}$ as the subset of production possibilities in which the firm does not require any cash. Formally,

$$\mathscr{Q}_0 = \left\{\mathbf{q} | \tilde{X}(\mathbf{q},\mathbf{d}_m) + \alpha I \geq 0\right\}. \tag{7}$$

It is immediate from Assumption (A3) and (5) that the function $\mathscr{L}(\mathbf{q})$ is non-decreasing in $\mathbf{q}$. The following lemma formalizes this statement.

Lemma 1 ($\mathscr{L}$ is Non-Decreasing in the Production Plan). *For any comparable pair of feasible production plans,* $\mathbf{q}_0, \mathbf{q}_1 \in \mathscr{Q}$, *with* $\mathbf{q}_0 \leq \mathbf{q}_1$ *we have* $\mathscr{L}(\mathbf{q}_0) \leq \mathscr{L}(\mathbf{q}_1)$.

Observe that $\tilde{X}(\mathbf{q},\mathbf{d}_m)$ is the minimum operating income obtained from production plan $\mathbf{q}$. Any production plan $\mathbf{q} \leq \mathbf{d}_m$ can be considered as pre-contracted production, and production plans $\mathbf{q} \geq \mathbf{d}_m$, generate stochastic operating income cash flows as $(\mathbf{q} - \mathbf{d}_m)^+$ is subject to demand fluctuations.

The operating income $\tilde{X}(\mathbf{q},\mathbf{d})$ is concave in $\mathbf{q}$ whenever $\tilde{R}(\mathbf{q},\mathbf{d})$ is concave in $\mathbf{q}$ and $C(\mathbf{q},\mathbf{d})$ is convex in $\mathbf{q}$. The following lemmas further characterize $\mathscr{Q}_0$ and $\mathscr{L}$.

Lemma 2 (Convexity of $\mathscr{L}$). If $C(\mathbf{q},\mathbf{d})$ *is convex (non-decreasing) on* $\{\mathbf{q}|\mathbf{q} \geq \mathbf{d}_m\}$, *then* $\mathscr{L}(\mathbf{q})$ *is a convex (non-decreasing) function.*

Proof. See Appendix A.

Lemma 3 (Convexity of $\mathscr{Q}_0$). *If* $\tilde{X}(\mathbf{q},\mathbf{d}_m)$ *is concave or quasi-concave on* $\{\mathbf{q}|\mathbf{q} \geq \mathbf{d}_m\}$, *then* $\mathscr{Q}_0$ *is a convex set.*

Proof. See Appendix A.

In the next section we discuss the notion of the value of the firm.

3.4. Firm (Stock) Value When the Firm Holds Riskless Capital

To assess the value of this single-period firm if a production plan $\mathbf{q}$ is adopted, the expected cash flow to investors $\mathrm{E}\tilde{F}(\mathbf{q}) = \mathrm{E}\tilde{X}(\mathbf{q}) + \alpha I + (1+r_f)\mathscr{L}(\mathbf{q})$ must be discounted back to time t. The appropriate discount rate has traditionally been estimated using the Capital Asset Pricing Model, or the APT (Ross 1976, Chamberlain and Rothschild 1983).

If stock returns are generated by a single exogenous factor e, generating shocks (henceforth referred to as the "market"), the APT implies that the value of the firm, $V(\mathbf{q})$, is given by

$$V(\mathbf{q}) = \frac{\mathrm{E}\tilde{F}(\mathbf{q}) - B(\mathbf{q})(r_e - r_f)}{1+r_f}, \tag{8}$$

where $\tilde{r}_e$ is the return of the market, and $r_e = \mathrm{E}\tilde{r}_e$. If we denote the variance of the market return as σ_e^2, the "cash flow beta," $B(\mathbf{q})$, is defined as

$$B(\mathbf{q}) = \frac{\mathrm{COV}\left(\widetilde{F}(\mathbf{q}), \tilde{r}_e\right)}{\sigma_{\tilde{e}}^2}. \quad (9)$$

Note that the risk-adjusted (certainty equivalent) cash flows in (8) are obtained by first adjusting the expected cash flows at $t+1$ with the firm's cash flow beta, and that the resulting risk-adjusted cash flows are then discounted back to time t using the risk-free interest rate r_f.

In the asset-pricing framework the relevant measure of risk is not the variability of the investors' cash flows, but the covariance of the cash flows with respect to the returns of the market.

3.5. Covariance Risk of the Cash Flows

To establish monotonicity (either positive or negative) of the covariance between the cash flows to investors and the returns on the market we assume, in this paper, that the random variables $(\mathbf{d}, \tilde{r}_e)$ are associated. In Appendix B, we discuss in detail why this assumption is necessary.[10]

For our purposes it is interesting to examine the normal probability distribution as it is often used in the operations management literature as an approximate model of demand, and in the finance literature as a model of returns. If we are concerned with a single product, and the joint density of demand and market returns is a bivariate normal distribution, then $(\mathbf{d}, \tilde{r}_e)$ are associated if the correlation coefficient is non-negative. In higher dimensions, if $(\mathbf{d}, \tilde{r}_e)$ are distributed according to a multivariate normal probability density, then $(\mathbf{d}, \tilde{r}_e)$ are associated if the off-diagonal elements of $-\Sigma^{-1}$ are all non-negative, where Σ is the covariance matrix of the joint probability distribution.

If we consider a $(n+1)$ vector of random variables $(\mathbf{d}, \tilde{r}_e)$, and if we assume that they are associated, then we can claim that the cash flow of the firm is positively correlated with the returns of the market if the operating income function, $\widetilde{X}(\mathbf{q}, \mathbf{d})$, is non-decreasing in demand for any $\mathbf{q}$. Lemma 4 below formalizes this statement.

LEMMA 4 (Correlation of Cash Flow and Market Returns). *If the returns on the market $\tilde{r}_e$ and demand $\mathbf{d}$ are associated, and if $\widetilde{X}(\mathbf{q}, \mathbf{d})$ is non-decreasing in demand for any $\mathbf{q}$ (i.e., Assumption (A1) holds), then the cash flow to investors, $\widetilde{F}$, resulting from any production plan $\mathbf{q}$ is positively correlated with the market returns, $\tilde{r}_e$.*

PROOF. See Appendix A.

Next, we explore the relationship between the production plan $\mathbf{q}$ and the covariance between the cash flows of the firm and the market returns, $\mathrm{COV}(\widetilde{F}(\mathbf{q}), \tilde{r}_e)$. Denote as $\psi(\mathbf{d}, \tilde{r}_e)$ the joint probability mass function of the demand $\mathbf{d}$ and the returns of the market $\tilde{r}_e$. Denote as ϕ^{ξ} and ϕ^{r} the marginal probability mass functions, and as Φ^{ξ} and Φ^{r} their respective distribution functions. Then, the covariance between the firm's cash flow $\widetilde{F}(\mathbf{q})$ and market returns $\tilde{r}_e$, $\mathrm{COV}(\widetilde{F}(\mathbf{q}), \tilde{r}_e) = \mathrm{E}[\widetilde{F}(\mathbf{q})\tilde{r}_e] - r_e\mathrm{E}\widetilde{F}(\mathbf{q})$ can be obtained as

$$\mathrm{COV}\left(\widetilde{F}(\mathbf{q}), \tilde{r}_e\right) = \int_{-\infty}^{+\infty}\int_{0}^{+\infty} (\tilde{r}_e - r_e)\widetilde{F}(\mathbf{q})\psi(\mathbf{d}, \tilde{r}_e)d\,\mathbf{d}\,d\tilde{r}_e. \quad (10)$$

LEMMA 5 (Monotonicity of Cash Flow Covariance in Production Plan). *If $(\mathbf{d}, \tilde{r}_e)$ are associated, and if Assumptions (A1) and (A2) are satisfied, then the covariance between the cash flows to investors, $\widetilde{F}(\mathbf{q})$, and market returns is non-decreasing in the production plan $\mathbf{q}$.*

PROOF. See Appendix A.

The following lemma further characterizes the covariance of the cash flows with the returns of the market as a function of $\mathbf{q}$. To this end we define the demand vector $\mathbf{d}_m$ as the vector whose elements are the minimum demand realizations. The vector $\mathbf{d}_m$ can be interpreted as the array constructed with pre-contracted orders at time t for all products i. Similarly, define the demand vector $\mathbf{d}_M$ as the vector of maximum demand realizations.

LEMMA 6 (Characterization of the Covariance of Cash Flow with Market Returns). *If Assumptions (A1) and (A2) are satisfied:*

(a) $\frac{\partial}{\partial q_i}\mathrm{COV}\left(\widetilde{F}(\mathbf{q}), \tilde{r}_e\right) \geq 0$ *for all i for all* $\mathbf{q} \geq \mathbf{d}_m$.
(b) $\frac{\partial}{\partial q_i}\mathrm{COV}\left(\widetilde{F}(\mathbf{q}), \tilde{r}_e\right) = 0$ *for* $\mathbf{q} \geq \mathbf{d}_M$.

PROOF. See Appendix A.

Collectively the three lemmas above describe how the covariance, $\mathrm{COV}\left(\widetilde{F}(\mathbf{q}), \tilde{r}_e\right)$, changes as a function of the production plan $\mathbf{q}$. Equation (9) implies that the cash flow beta B is zero for $0 \leq \mathbf{q} \leq \mathbf{d}_m$. It is then non-decreasing in each q_i until reaching $\mathbf{d}_M$, and is constant for all larger $\mathbf{q}$. The implication is that in general neither $B(\mathbf{q})$ nor $\mathrm{COV}\left(\widetilde{F}(\mathbf{q}), \tilde{r}_e\right)$ can be characterized as either concave or convex in $\mathbf{q}$, except when $\mathbf{d}$ is independent of $\tilde{r}_e$ and, in this trivial case, $\mathrm{COV}\left(\widetilde{F}(\mathbf{q}), \tilde{r}_e\right) = B(\mathbf{q}) = 0$ for all values of $\mathbf{q}$.

3.6. Shareholders' Total Wealth in a Mixed Contracts Economy

If the firm's manager undertakes production plan $\mathbf{q}$, he will require a fixed investment of I plus a risk-free cash investment of $\mathscr{L}(\mathbf{q})$. When these investments are made, the firm creates a value of $V(\mathbf{q})$. Thus the NPV created inside the firm, as a function of $\mathbf{q}$, is given by

$$NPV^F(\mathbf{q}) = V(\mathbf{q}) - I - \mathscr{L}(\mathbf{q}) = \frac{\mathrm{E}\widetilde{X}(\mathbf{q}) + \alpha I - B(\mathbf{q})(r_e - r_f)}{1 + r_f} - I. \tag{11}$$

Since $\widetilde{F}(\mathbf{q})$ and $\widetilde{X}(\mathbf{q})$ differ only by a constant (i.e., $(1+r_f)\mathscr{L}(\mathbf{q})$) for all demand outcomes, we have $\mathrm{COV}(\widetilde{F}(\mathbf{q}), \tilde{r}_e) = \mathrm{COV}(\widetilde{X}(\mathbf{q}), \tilde{r}_e)$. The cash flow beta, $B(\mathbf{q})$, is affected by the operating cash flow, $\widetilde{X}(\mathbf{q})$, but it is otherwise independent of the firm's cash requirement, $\mathscr{L}(\mathbf{q})$. Thus, *it follows from* (11) *that the NPV* created inside the firm, $NPV^F(\mathbf{q})$, is unaffected by the firm's cash requirements.

In a mixed contracts economy there are potentially positive NPV investment opportunities in private businesses, and to allow for this expanded set of opportunities we distinguish between NPV^F, the NPV created inside the firm, and NPV^O, the NPV that the shareholders can create outside the firm. The shareholders' *total* wealth thus depends on both NPV notions.

To understand the trade-offs involved in the manager's decision to maximize the shareholders' total wealth, we define the productivity of capital as the NPV created per dollar of capital invested. For any non-zero production plan $\mathbf{q}$, the productivity of the firm's invested capital, $\eta(\mathbf{q})$, is

$$\eta(\mathbf{q}) = \frac{NPV^F(\mathbf{q})}{\mathscr{L}(\mathbf{q}) + I}. \tag{12}$$

Our analysis assumes that to capture a fraction of NPV^F, an investor must be a shareholder at $t = 0$. If the manager can create the same NPV^F with less invested capital (e.g., the manager could use risk management activities to free up $\Delta\mathscr{L}$ in cash without changing NPV^F), then shareholders will capture a larger fraction of the NPV^F per dollar of their investment in the firm. Any freed-up capital that is not required can be invested by the shareholders by taking positive NPV ownership positions in private businesses. That is, we assume that the productivity of capital outside the securities markets, η_0, is strictly positive, and we further assume that η_0 is equal for all of the firm's shareholders. We emphasize that $\eta_0 > 0$ cannot be rationalized in models where all economic activity occurs only within the securities markets.[11]

With the investors' opportunity set thus expanded, it is no longer clear that decisions that maximize NPV^F, the NPV inside the firm, also maximize the shareholders' wealth. To maximize shareholders' total wealth, we need to also consider their wealth-creation opportunities outside the securities markets, $NPV^O(\mathbf{q})$.

If the working capital plus investment required by operating plan $\mathbf{q}$, $I\mathbf{1}_{\{\mathbf{q}\geq 0\}} + \mathscr{L}(\mathbf{q})$,[12] were invested outside the firm, it would generate

$$NPV^O(\mathbf{q}) = \eta_0\left(I\mathbf{1}_{\{\mathbf{q}\geq 0\}} + \mathscr{L}(\mathbf{q})\right). \tag{13}$$

The function $\mathbf{1}_{\{\mathbf{q}\geq 0\}}$ is an indicator function taking a value of 1 if $\mathbf{q} \geq 0$ and zero if $\mathbf{q} = 0$. Thus, if $\mathbf{q} = 0$, $NPV^O(0) = 0$ as $\mathscr{L}(0) = 0$. That is, if the firm is liquidated at time t it requires no investment and the shareholders do not forfeit any outside wealth creation opportunities. In this context, the net increase in shareholders' wealth under a production plan $\mathbf{q}$, denoted as $W(\mathbf{q})$, is the NPV created inside the firm, $NPV^F(\mathbf{q})$, minus the NPV that shareholders could create on their own outside the stock markets, $NPV^O(\mathbf{q})$. Hence, the production plan that maximizes shareholders' wealth, denoted as $\mathbf{q}^W$, can be obtained by solving the following program:

$$W^* = W(\mathbf{q}^W) = \max_{\mathbf{q}\in\mathscr{Q}}\{NPV^F(\mathbf{q}) - NPV^O(\mathbf{q})\} = \max_{\mathbf{q}\in\mathscr{Q}}\left\{V(\mathbf{q}) - (1+\eta_0)\left(I\mathbf{1}_{\{\mathbf{q}\geq 0\}} + \mathscr{L}(\mathbf{q})\right)\right\}. \tag{14}$$

4. The Effect of Altering a Production Plan on Shareholders' Total Wealth

In a mixed contracts economy, the shareholders do not benefit from every positive-NPV project inside the firm. Projects undertaken by the firm carry an opportunity cost of the capital that depends on the wealth-creation opportunities outside the securities markets. If this opportunity cost exceeds the project's NPV, it is in the shareholders' best interests if these projects are not pursued. We denote as $\mathbf{q}^N$ the production plan maximizing NPV^F. Theorem 1, below, by comparing the relative magnitudes of $\mathbf{q}^N$ and the shareholders' total wealth-maximizing production plan, $\mathbf{q}^W$, establishes this intuition.

Theorem 1 (Maximizing the Firm's NPV may not Maximize Shareholders' Wealth). *If $\frac{\partial}{\partial q_i^N}\mathscr{L}(\mathbf{q}^N) > 0$ for any product i, then the production plan that maximizes the shareholders' total wealth is smaller than or equal to the plan that maximizes NPV^F, $\mathbf{q}^W \leq \mathbf{q}^N$. Conversely, if $\frac{\partial}{\partial q_i^N}\mathscr{L}(\mathbf{q}^N) = 0$, for all products i, the production plan maximizing shareholders' wealth also maximizes NPV^F, $\mathbf{q}^W = \mathbf{q}^N$.*

Proof. See Appendix A.

There is a different way to state this theorem. If the firm's NPV-maximizing production plan requires no working capital, $\mathbf{q}^N \in \mathscr{Q}_0$, it maximizes the shareholders' wealth, that is, $\mathbf{q}^W = \mathbf{q}^N$. However, if the

NPV-maximizing production plan requires working capital, $\mathbf{q}^W \leq \mathbf{q}^N$.

Thus, in evaluating the transition from an operating plan $\mathbf{q}$ to a different operating plan $\mathbf{q}'$, the manager must consider not only the change in wealth created inside the firm, but also the implications for the shareholders' wealth of any resulting change in working capital requirements. Below we state this formally. This finding has profound implications for the practice of operations management, as the subsequent discussion and numerical examples will show.

Corollary 1 (Justifying Changes in the Production Plan). *Changing a production plan* $\mathbf{q}$ *with working capital requirements* $\mathscr{L}(\mathbf{q})$ *to a different production plan* $\mathbf{q}'$ *with working capital requirements of* $\mathscr{L}(\mathbf{q}')$ *increases the shareholders' wealth if and only if the wealth increase inside the firm,* $NPV^F(\mathbf{q}') - NPV^F(\mathbf{q})$, *exceeds the opportunity cost of the increased working capital investment,* $\eta_0(\mathscr{L}(\mathbf{q}') - L(\mathbf{q}))$.

This presents us with two different ways to create shareholders' wealth: (a) we can increase NPV^F more productively than the shareholders' investments outside the security markets, or (b) we can decrease NPV^F if the freed working capital will result in a net increase in shareholders' wealth. This is an important result. Unlike in the NPV-maximization models of the traditional literature, the cash requirement, through its impact on the firm's productivity, has an effect on the choice of the optimal operating plan.

Corollary 2 highlights the relevance of using capital productively in the firm.

Corollary 2 (Positive Firm NPV is not Sufficient for Financial Viability). *If the firm's productivity,* $\eta(\mathbf{q})$, *associated with any feasible production plan is lower than the productivity of investments outside the security markets* η_0, *setting* $\mathbf{q}^W = 0$ *and liquidating the firm is optimal for the shareholders.*

We next illustrate the implications of the standard securities economy analysis, and of the mixed contracts economy analysis, by examining three production alternatives under demand uncertainty. Unlike the case of NPV maximization in the traditional model, the cash requirement, $\mathscr{L}(\mathbf{q})$, now has an effect on the selection of the production plan that maximizes shareholders' wealth.

Example 3 (Evaluation of Production Alternatives). Consider a single-product firm for which the joint probability distribution of demand with the market returns is as shown in Table 1. The firm is considering three alternative production systems. The first, an MTS system (see Example 1), requires an initial investment, I at time t of \$60. The variable cost of production is $c = \$1$/unit. The next alternative, a *Fast-Response* MTO system (see Example 2), requires a larger investment, $I = \$75$. It has a capacity, $q^h = 100$, and a larger variable cost $c = \$1.6$/unit. In relation to the MTS system, the MTO process eliminates overstocking/understocking risks, but it increases the required investment and unit production cost. Under the third alternative, labeled *Asset-Light* MTO, the firm outsources production. This lowers the initial investment but increases the unit production cost. Assume that this alternative has $I = \$32.23$, and $c = \$1.80$/unit. In all three cases, the sale price is $p = \$2$, and the firm recovers 75% of the investment at $t+1$, $\alpha = 0.75$. Table 1 also contains information on the firm's operating income for the three production systems. Table 2 summarizes the relevant variables for the subsequent discussion.

If $\mathbf{q} = 100$, the expected cash flow from operations in the MTS system, from Table 1, are $E\tilde{X} = \$28.0$. If demand is $\xi = 10$, the cash flow from operations plus with the salvage value of the initial investment is $-\$80 + (0.75)\$60 = -\$35$. From Equation (4) the manager must invest $\mathscr{L} = \$35/(1 + r_f) = \$34$ in a risk-free security at the beginning of the time period. The investors' expected cash flow at the end of the period is, from (6), $E\tilde{F} = E\tilde{X} + (1 + r_f)\mathscr{L} + \alpha I = \$28 + \$35 + \$45 = \$108$. Using the information in Table 1 and Equation (9) the cash flow beta is $B = 75$ and, from (8), we obtain $V = \$96.86$. Hence the NPV created inside the firm is $NPV^F = V - \mathscr{L} - I = \$96.86 - \$34 - \$60 = \$2.86$. The NPV that the shareholders could have obtained outside the security markets (with the investment required under this production alternative) is $NPV^O = \eta_0(\mathscr{L} + I) = 0.035(\$34 + \$60) = \$3.29$. Since this NPV^O is the shareholders' opportunity cost of the capital, this production alternative, although it has a positive NPV inside the firm, actually reduces the shareholder's wealth by $\Delta W = -\$0.43$.

The calculations yielding the remaining rows in Table 2 are similar. It can readily be seen from the last

Table 1 Joint Probabilities, Operating Cash Flows, and Summary Information

	Market Returns		Operating Cash Flows $\tilde{X}$			
			MTS System		Fast-Response	Asset-Light
Demand (ξ)	$\tilde{r}_e = -0.1$	$\tilde{r}_e = 0.3$	$\mathbf{q} = 10$	$\mathbf{q} = 100$	MTO	MTO
100	0.2	0.4	10	100	40	20
10	0.2	0.2	10	−80	4	2
$\bar{r}_e = 0.14$	$\rho = 0.17$					
$\sigma_e^2 = 0.038$	$r_f = 0.03$					

Table 2 Summary of Financial Evaluation of Projects

	Evaluation of Changes in Shareholders' Wealth									
	$E\tilde{X}$	$\mathscr{L}$	$E\tilde{F}$	I	B	V	NPV^F	NPV^O	η	ΔW
MTS System ($\mathbf{q} = 10$)	10.0	0	55.0	60.00	0.0	53.4	− 6.60	2.10	− 11.0%	− 8.70
MTS System ($\mathbf{q} = 100$)	28.0	34	108.0	60.00	75.0	96.9	2.86	3.29	3.0%	− 0.43
Fast-Response MTO	25.6	0	81.9	75.00	15.0	77.9	2.86	2.63	3.8%	0.24
Asset-Light MTO	12.8	0	37.0	32.23	7.5	35.1	2.86	1.13	8.9%	1.74

The calculations of NPV^O and ΔW assume $\eta_0 = 3.5\%$.
The asset recovery factor $\alpha = 0.75$ for all three cases.

column in Table 2 that, for the MTS system, the production quantity maximizing NPV^F is $\mathbf{q}^N = 100$. In fact, for the MTS System the shareholders' wealth-maximizing production quantity is zero, $\mathbf{q}^W = 0$; it is best for the shareholders if the firm is liquidated.

Since the *Fast-Response* and *Asset-Light* alternatives are MTO production systems, we have $\mathbf{q} = \xi$ for both. To see what is involved, note that the MTS system using $\mathbf{q}^N = 100$ and the two MTO alternatives have an identical NPV^F meaning that, in a securities-only economy, all three would be equivalent. The fact that the *Asset Light* production alternative generates the same wealth inside the firm as the other two alternatives that require only a fraction of the initial investment would be altogether inconsequential.

In the mixed-contracts economy, the NPV that investors could have earned outside the securities markets, NPV^O, is obtained from (13). The production system that maximizes the shareholders' total wealth is the *Asset-Light* MTO. □

The above comparison of the MTS and MTO systems shows that the shareholders' wealth implications of production delays are complex. In an MTO system, we assume all production is initiated and delivered after demand is observed. This eliminates inventory risk, but not the demand risk. Even if production and delivery are instantaneous, the firm's cash flows have financial risk induced by demand variability, and in our model, the firm may have to hold cash. To see why, consider that in addition to materials and other variable costs, a firm needs infrastructure; it incurs fixed operating expenses as part of its activities (see Lederer and Singhal 1988). Even if suppliers deliver the right quantities of materials instantly if and when demand is materialized, and even if production and delivery are instantaneous, there is still the possibility that demand will not be large enough to generate the cash flows to pay the factors providing infrastructure and generating the fixed operating expenses.

The calculations of the MTS System can be reworked to illustrate that since the cash requirement, $\mathscr{L}$, is invested at the risk-free interest rate, the wealth created inside the firm, NPV^F, is insensitive to $\mathscr{L}$. This should not be surprising as risk-free cash is a zero-NPV investment. However, cash requirements increase the investment required and reduce the productivity of capital of the firm. In this sense, hedging practices that reduce the firm's cash requirements can increase the shareholders'wealth by freeing up capital that shareholders can use to invest outside the securities markets. We investigate this implication in the next section.

5. Implications of Financial Hedging for Operating Policies

Hedging, both financial and operational, has been extensively studied in the operations management literature as a mechanism for reducing the risk of the firm's cash flows. In this section, we consider the possibility of using a financial hedge to modify the cash flows to investors $\tilde{F}$, and examine its effect on the optimal production plan and on firm value.

We denote as $\tilde{X}^H$ the cash flow from the financial hedge, and as $\tilde{F}^H$ the cash flows to investors in the hedged firm. We denote as $\mathscr{L}^H$ the cash requirements of the hedged firm. If we allow the hedging policy (e.g., the mix of transactions and volumes) to change as a function of the production plan $\mathbf{q}$, the $t+1$ cash flow to the investors of the hedged firm is given by

$$\tilde{F}^H(\mathbf{q}) = \tilde{X}(\mathbf{q}) + (1+r_f)\mathscr{L}^H(\mathbf{q}) + \alpha I + \tilde{X}^H(\mathbf{q})$$
$$= \tilde{F}(\mathbf{q}) + \tilde{X}^H(\mathbf{q}) + (1+r_f)(\mathscr{L}^H(\mathbf{q}) - \mathscr{L}(\mathbf{q})). \quad (15)$$

However, to obtain these cash flows, the firm will incur the cost of the financial hedge at time t. We denote as C^H the cost of the financial hedge, and as $V^H(\mathbf{q})$ the value at time t of $\tilde{F}^H(\mathbf{q})$. The NPV created *inside* the hedged firm, NPV^H, is given by

$$NPV^H(\mathbf{q}) = V^H(\mathbf{q}) - (I + \mathscr{L}^H(\mathbf{q}) + C^H(\mathbf{q})) \quad (16)$$

If the financial hedge is priced correctly, the cost of the hedge at time t is $C^H = E^{\mathbb{Q}}\tilde{X}^H$, where the notation $E^{\mathbb{Q}}$ indicates that the expectation is taken over a *risk-neutral* (martingale) probability measure.

Since the value of the cash flows of the financial hedge is equal to its cost C^H, and since the cash flows from the hedge and the cash flows of the firm are both priced correctly, the time t value of the financially hedged firm is

$$V^H(\mathbf{q}) = V(\mathbf{q}) + C^H(\mathbf{q}) + \mathscr{L}^H(\mathbf{q})\mathscr{L}(\mathbf{q}). \quad (17)$$

From (17) and (16) it follows that the NPV of the financially hedged firm, $NPV^H(\mathbf{q})$, is identical to the NPV created by the unhedged firm, $NPV^F(\mathbf{q})$, for any operating policy $\mathbf{q}$.

$$\begin{aligned} NPV^H(\mathbf{q}) &= V^H(\mathbf{q}) - \left(I + \mathscr{L}^H(\mathbf{q}) + C^H(\mathbf{q})\right) \\ &= V(\mathbf{q}) - I - \mathscr{L}(\mathbf{q}) = NPV^F(\mathbf{q}). \end{aligned} \quad (18)$$

This result should not be surprising. The change in the value of the firm arising from the hedge is exactly offset in the NPV calculation by the cost of the hedge, and changes in corporate cash holdings do not affect the NPV created inside the firm. However, as we show next, if the hedge reduces the firm's *total* cash requirements (the change in the necessary cash *net* of the cost of the hedge), it increases the shareholders' total wealth and, moreover, that it can alter the firm's optimal operating plan.

THEOREM 2 (Hedging Increases Shareholders' Wealth). *If (a) the productivity of investments outside the securities markets is positive, $\eta_0 > 0$, and (b) a financial hedge can be selected so that $\mathscr{L}^H(\mathbf{q}) + C^H(\mathbf{q}) \le \mathscr{L}(\mathbf{q})$, then (1) the cash released increases shareholders' total wealth. Moreover, if $\frac{\partial}{\partial q_i}\mathscr{L}^H(\mathbf{q}^W) + \frac{\partial}{\partial q_i}C^H(\mathbf{q}^W) \le \frac{\partial}{\partial q_i}\mathscr{L}(\mathbf{q}^W)$, (i.e., the marginal increase in the hedged firm's cash requirements is less than that for the unhedged firm) then (2) the shareholders' wealth can be further increased by increasing the production plan from $\mathbf{q}^W$ to $\mathbf{q}^H$, with $\mathbf{q}^W \le \mathbf{q}^H \le \mathbf{q}^N$.*

PROOF. See Appendix A

Hedging can have four effects on shareholders' wealth and on the firm's competitiveness. First, by reducing the firm's working capital requirements, it can increase the shareholders' total wealth. Second, it increases the cash-free set, $\mathscr{Q}_0$, and potentially increases the firm's ability to increase NPV^F by increasing $\mathbf{q}$. Third, if by reducing working capital requirements $\mathbf{q}^N \in \mathscr{Q}_0$, the hedged firm can operate at its full NPV-maximizing potential, $\mathbf{q}^W \le \mathbf{q}^H = \mathbf{q}^N$. Finally, by increasing the productivity of the firm's capital, hedging increases the firm's long-term survival.[13]

Even when a hedge results in an increase in production, the optimal operating plan will never be larger than the NPV-maximizing plan.

The magnitude of the benefit derived from hedging hinges on the ability of the firm's manager to tailor the financial hedge to reduce the amount of cash needed from $\mathscr{L}(\mathbf{q})$ down to a $\mathscr{L}^H(\mathbf{q}) < L(\mathbf{q})$. This will happen only if the cash flow of the financial hedge $\tilde{X}^H$ is guaranteed to be positive for the demand outcomes at and close to $\mathbf{d}_m$.

The following example illustrates the two different ways that financial hedging can increase shareholders' wealth. First, by lowering "downside risk," it can reduce the firm's cash requirements and hence its opportunity cost of capital NPV^O. Second, by reducing the cash requirements it increases the firm's productivity and, as Theorem 2 suggests, it allows the firm to operate at a higher level of output. This increases NPV^F.

EXAMPLE 4 (Impact of Hedging on Shareholders' Wealth). Consider a firm producing a single-product using an MTS production system (see Example 1). The joint probability distribution of demand with the market returns is as shown in Table 3. The product's sale price is \$2 per unit, and the variable cost of production is \$1 per unit. No initial investment is required, $I = 0$. Table 3 also contains information on the product's demand and the firm's operating income. There is a traded security, $\tilde{S}$, whose payoffs are correlated with the demand of the product as shown in Table 3. We assume that a *put* option on security $\tilde{S}$, with a strike price of \$20 is also traded. This option's stochastic payoffs are shown in Table 3 in the column labeled $\tilde{P}(20)$.

The Unhedged Firm. The analysis of the unhedged cash flows is similar to the illustrated in Example 3. In this example, however, since $I = 0$, the minimum operating cash flow is $-\$80$ when $\mathbf{q} = 100$. We thus have $\mathscr{L}(100) = 80/(1 + r_f) = \$76.2$. The remaining calculations are reported in Table 4, and they reveal that although the NPV^F-maximizing production plan, $\mathbf{q}^N = 100$, creates \$20.2 of NPV inside the firm, it increases the shareholders' total wealth by only \$8.8. The shareholders' total wealth-maximizing production plan is $\mathbf{q}^W = 10$. Although this plan creates lower NPV^F, it results in a total wealth increase of \$9.5.

The Hedging Plan. The hedging plan consists of buying n units of $\tilde{P}(20)$. To value the impact of using the put, $\tilde{P}(20)$, as a hedge, we must first calculate its price. To this end, we first use Equations (9) and (8) and obtain the value of security $\tilde{S}$, as \$14.88. If we denote as π_{100} and π_{10} the *risk-neutral* probabilities

Table 3 Summary Information of Operating and Hedging Cash Flows

	Market Returns					
Demand (ξ)	$\tilde{r}_e = -0.1$	$\tilde{r}_e = 0.3$	$\tilde{X}(\mathbf{q} = 100)$	$\tilde{X}(\mathbf{q} = 10)$	$\tilde{S}$	$\tilde{P}(20)$
100	0.2	0.4	100	10	20	0
10	0.2	0.2	−80	10	10	10

$\bar{r}_e = 0.14$.
$\sigma_e^2 = 0.038 r_f = 0.05$.

Table 4 Evaluation of Unhedged Production Plans

Production Plan	$\widetilde{EX}$	$\mathscr{L}$	$\widetilde{EF}$	B	V	NPV^F	NPV^O	ΔW
$\mathbf{q} = 10$	10.0	0.0	10.0	0.0	9.5	9.5	0.0	9.5
$\mathbf{q} = 100$	28.0	76.2	104.2	75.0	96.4	20.2	11.4	8.8

The calculations above assume $\eta_0 = 15\%$.

associated with the demand outcomes of 100 and 10, respectively, it follows from the information in Table 3 that the risk-neutral probabilities must satisfy $(\$20\pi_{100} + \$10\pi_{10})/(1 + r_f) = \$14.88$, and $\pi_{100} + \pi_{10} = \$1$. Solving these equations yields $\pi_{100} = 0.5624$, and $\pi_{10} = 0.4376$. These probabilities imply that the price of the put $\widetilde{P}(20)$, denoted as V^P, is $V^P = (\$0\pi_{100} + \$10\pi_{10})/(1 + r_f) = \$4.17$.

The Hedged Firm. In Table 5, we summarize the relevant variables. Where applicable, the numbers in parentheses under the column headings refer to the equations used to define the corresponding variable. The expected cash flow from operations $\widetilde{EX}$ is obtained as in Example 3. The expected cash from the hedge can be obtained from the information in Table 3. For example, if we buy 8 units of the put, $n = 8$, and the expected cash flow is $\widetilde{EX}^H = 8(0.6 \times \$0 + 0.4 \times \$10) = \$32$.

Under plan $\mathbf{q} = 10$, as Tables 3 and 4 show, the unhedged firm requires no cash, $\mathscr{L}(10) = 0$. Hence, we select $= 0$ (since, as seen earlier, no hedging plan can create wealth for the shareholders in this case). However, for $\mathbf{q} = 100$, we have $\mathscr{L}(100) = 80/(1 + r_f)$, and each additional unit of $\widetilde{P}(20)$ that we buy (up to $n = 8$) eliminates the need to hold $\$10/(1 + r_f)$ in cash (since the put pays off \$10 in the earnings shortfall state). However, each unit of $\widetilde{P}(20)$ costs \$4.17; thus, a hedging plan with $n = 8$ drives $\mathscr{L}^H$ to zero, but it requires an initial investment of $8 \times \$4.17 = \$33.36$.

Under the plan $\mathbf{q} = 100$ and $n = 8$, the expected value $\widetilde{EF}^H$ is obtained as $\widetilde{EF}^H = \widetilde{EX} + \widetilde{EX}^H + (1 + r_f)\mathscr{L}^H = \$28.0 + \$32.0 + \$0 = \$60.0$.

The value of the hedged firm V^H is obtained from Equation (17) as $V^H = V + C^H + \mathscr{L}^H - \mathscr{L}$. For the case with $\mathbf{q} = 100$ we obtain from Table 4 $V = \$96.4$ and $\mathscr{L} = \$76.2$; since the cost of $\widetilde{P}(20)$ is \$4.17 per unit, for $n = 8$ we have $C^H = 8 \times \$4.17 = \$33.36$. Therefore, the value of the hedged firm is $V^H = \$96.4 + \$33.36 + \$0 - \$76.2 = \$53.56$. To obtain the NPV created inside the hedged firm we need to subtract from the value of the firm, V^H, the cost of the financial hedge, C^H, in addition to the initial investment and cash requirements. Thus, $NPV^H = \$53.56 - \$33.36 - \$0 - \$0 = \$20.2$.

The Total Wealth Increase from Hedging. The opportunity cost of the total capital invested, NPV^O, is $NPV^O = \eta_0(I + \mathscr{L}^H + C^H) = 0.15 \times (\$0 + \$33.36 + \$0) = \$5.0$. Thus, for the $\mathbf{q} = 100$ and $n = 8$ cases we obtain $\Delta W = NPV^F - NPV^O = \$20.2 - \$5.0 = \$15.2$. This is shown in the last column of Table 5.

Comparing ΔW in Tables 4 and 5 shows that for the unhedged firm we have $\mathbf{q}^W = 10$ with $\Delta W = \$9.5$. For the hedged firm, we have $\mathbf{q}^H = 100$ leading to $\Delta W = \$15.2$. Thus, the total shareholder wealth created by hedging is $\$15.2 - \$9.5 = \$6.7$.

Two Wealth Effects. It is instructive to decompose the total wealth gains into two effects:

(a) Change in the opportunity cost of capital NPV^O: The change in the opportunity cost of capital arising from hedging is the product of η_0 times the total capital released corresponding to the best unhedged and hedged production plans, $\mathbf{q}^W$ and $\mathbf{q}^H$, respectively. This is obtained as $\eta_0(\mathscr{L} - \mathscr{L}^H - C^H) = 0.15(\$0 - \$0 - \$33.36) = -\$5.0$. Since the capital requirements actually increased, there is a net loss of \$5.0.

(b) Change in the wealth created by the production plan, NPV^F: However, by hedging the cash flows, the firm's productivity under $\mathbf{q} = 100$ and $n = 8$ increases, and the optimal production plan increases from $\mathbf{q}^W = 10$ to $\mathbf{q}^H = 100$. The increase in NPV^F, from Table 4, is $NPV^F(\mathbf{q}^W = 10) = \$9.5$. From Table 5 we have $NPV^F(\mathbf{q}^H = 100) = \$20.2$. Since $NPV^F = NPV^H$ for any production plan, the increase in NPV resulting from the change in production plans is $NPV^H(\mathbf{q}^H = 100) - NPV^F(\mathbf{q}^W = 10) = \$20.2 - \$9.5 = \$12.7$.

Combining the increase in the opportunity cost of capital with the gains in the NPV created inside the firm, we obtain $-\$5.0 + \$12.7 = \$6.7$.

6. Contextual Positioning in the Operations Management Literature

If we assume the firm operates in a securities economy, the productivity of capital becomes irrelevant, and our results in the previous two sections must be modified. As noted, it is not possible for any shareholder to find positive-NPV investments outside the firm. In our model, this is equivalent to assuming $\eta_0 = 0$. If $\eta_0 = 0$, our model yields results that are well known in the extant literature.

Table 5 Evaluation of Hedged Production Plans

Production Plan	Hedging Plan	$\widetilde{EX}$	$\widetilde{EX}^H$	$\mathscr{L}^H$	$\widetilde{EF}^H$ (15)	V^H (17)	NPV^H (16)	NPV^O	ΔW
$\mathbf{q} = 10$	$n = 0$	10.0	0.0	0.0	10.0	10.0	9.5	0.0	9.5
$\mathbf{q} = 100$	$n = 8$	28.0	32.0	0.0	60.0	53.6	20.2	5.0	15.2

If $\eta_0 = 0$, it follows from (13) that $NPV^O = 0$, and the problem of identifying the shareholders' wealth maximizing plan, $\mathbf{q}^W$, formulated earlier in (14), reduces to

$$W^* = W(\mathbf{q}^W) = \max_{\mathbf{q}\in\mathcal{Q}} NPV^F(\mathbf{q}) = \max_{\mathbf{q}\in\mathcal{Q}}\{V(\mathbf{q}) - I\mathbf{1}_{\{\mathbf{q}\geq 0\}} - \mathcal{L}(\mathbf{q})\}. \quad (19)$$

Thus assuming $\eta_0 = 0$ yields the well-known result in finance that decisions that maximize NPV^F maximize the shareholders' wealth; $\mathbf{q}^W = \mathbf{q}^N$.

Note that if $\eta_0 = 0$, it follows from the First Order Conditions (see equation (A2) in the proof of Theorem 1 in Appendix A) that the firm's cash requirements $\mathcal{L}(\mathbf{q})$ play no role in the selection of the optimal production plan. This follows from the observation that corporate cash, $\mathcal{L}(\mathbf{q})$, invested at the risk-free interest rate produces zero NPV. Thus, from the perspective of maximizing NPV^F, the selection of the optimal production plan, $\mathbf{q}^W = \mathbf{q}^N$, is unaffected by $\mathcal{L}(\mathbf{q})$.

Operations Management researchers Singhal (1988) and Lederer and Singhal (1988), among others, have studied the selection of the production plan maximizing the NPV created inside the firm, NPV^F. By setting $\eta_0 = 0$ in our model, it is possible to replicate their results.

In section 5, we justified financial hedging as a means of increasing the shareholders' wealth by reducing the firm's cash requirements. Equation (18) indicates that the NPV created by a financially hedged firm equals the NPV created without the financial hedge for any production plan $\mathbf{q}$. Thus if $\eta_0 = 0$, and hence $NPV^O = 0$, financial hedging cannot create any shareholders' wealth. Moreover, Equation (18) implies that the optimal production plan is unaffected by the use of a financial hedge, so that $\mathbf{q}^H = \mathbf{q}^W = \mathbf{q}^N$. This is a well-known result in the finance theory.

Conversely, the results of sections 4 and 5 cannot be obtained from models in the extant literature.

7. Concluding Remarks

In a mixed contracts economy the shareholders' total wealth depends on two effects –on the wealth created within the firm, NPV^F, by the firm's investment plan and on the investment plan's productivity, defined as the wealth created in the firm, NPV^F, per dollar of capital that is needed to support the investment. This definition of productivity is rich in the sense that it captures the economic implications of interest rates and the risk premiums (these are implicit in the market's valuation of the investment plan's cash flows). Productivity is important in a mixed contracts economy because the capital required to generate these cash flows depends not only on the cost of the investment (e.g., machinery) but also on the amount of riskless working capital that is required to optimally provide the owners of non-traded contracts their opportunity costs.

The numerical examples in sections 3 and 4 are the first in the literature to show, within the valuation framework, that the productivity of capital is relevant for shareholders' wealth. Moreover, Example 4 shows that even without the usual market frictions financial hedging has shareholders' wealth implications, and that it can affect the optimal production plan.

As Examples 3 and 4 illustrate, production and operations management practices that can lower the firm's working capital requirements (e.g., compare the MTO system with the *Rapid-Response* MTO in Example 3) and thus boost the firm's productivity. Similarly, operating practices, such as outsourcing, that generate more NPV per dollar invested (compare the *Rapid-Response* MTO with the *Asset-Light* MTO system in Example 3), increase productivity. To the extent that operations management and financial risk management (hedging) can increase the firm's productivity, they increase the firm's ability to survive in a competitive environment. Moreover, by reducing the amount of capital that shareholders must invest within the firm, operations and risk management practices allow the firm's investors to further augment their wealth by pursuing wealth-creating opportunities outside the markets.

Our results can be integrated seamlessly with real options approaches. The underlying principle in the real options approach is that if the cash flows of a project can be replicated through a portfolio of traded securities, the value of the project is the value of the portfolio (a no-arbitrage argument). In this paper, we value the cash flows using the APT. As a no-arbitrage approach, our analysis is entirely consistent with the real options approach. If the firm's (or project's) cash flows (including the cash requirements $\mathcal{L}$) can be defined in terms of a set of traded securities (e.g., stocks, bonds, or a traded contracts on commodities), they can also be valued using the real options approach. In this case, more complex, dynamic, operating decisions can be studied. However, since operating risk will have to be evaluated period-by-period, the cash requirements must be addressed individually for each period. Even though the optimal policies may not be myopic, the nature of our research findings will not change. The difficulty (limitation) in using the real options approach arises not from within our model, but from the challenge of finding a portfolio that replicates $\widetilde{X}(\mathbf{q})$.

In general, if we acknowledge the existence of investments with positive NPV outside the security markets, then $\eta_0 > 0$. In this case, as we have shown, operations and risk management practices that lower working capital needs increase the shareholders' wealth beyond what is captured by NPV^F. However, to obtain specific shareholders' wealth-maximizing operating policies, $\mathbf{q}^W$, we require more information about the magnitude of η_0. Further research is needed in this direction.

Appendix A: Proofs

PROOF OF LEMMA 2: Observe that $\widetilde{R}(\mathbf{q}, \mathbf{d}_m)$ is the constant revenue generated by supplying $\mathbf{d}_m$. Hence $-\widetilde{X}(\mathbf{q}, \mathbf{d}_m)$ is convex (non-decreasing) whenever C is convex (non-decreasing). The result is immediate from (5), and the fact that $\max(x, 0) = (x)^+$ is convex and non-decreasing. □

PROOF OF LEMMA 3: Immediate since $\mathcal{Q}_0$ is by definition (7) a level set, specifically the zero level set, of $-\widetilde{X}(\mathbf{q}, \mathbf{d}_m) - \alpha I$ which is, by assumption, a convex or quasi-convex function. □

PROOF OF LEMMA 4: Immediate from Definition 1 in Appendix B. If $f(\mathbf{d}) = \widetilde{F}$ and $g(\tilde{r}_e) = \tilde{r}_e$, then both f and g are non-decreasing in $(\mathbf{d}, \tilde{r}_e)$; hence $\widetilde{F}$ and $\tilde{r}_e$ are positively correlated. □

PROOF OF LEMMA 5: We need to show that the partial derivative of $\mathrm{COV}(\widetilde{F}(\mathbf{q}), \tilde{r}_e)$ with respect to q_i is non-negative for all i. From (10) above we obtain

$$\frac{\partial}{\partial q_i}\mathrm{COV}\left(\widetilde{F}(\mathbf{q}), \tilde{r}_e\right) = \int_{-\infty}^{+\infty}\int_0^{+\infty}\frac{\partial}{\partial q_i}\widetilde{F}(\mathbf{q})(\tilde{r}_e - r_e)\psi(\mathbf{d}, \tilde{r}_e)d\,\mathbf{d}\,d\tilde{r}_e. \quad (A1)$$

For each product i we can define functions $f(\mathbf{d}) = \frac{\partial}{\partial q_i}\widetilde{F}(\mathbf{q})$ and $g(\tilde{r}_e) = \tilde{r}_e - r_e$, and we can write $\frac{\partial}{\partial q_i}\mathrm{COV}(\widetilde{F}(\mathbf{q}), \tilde{r}_e) = \mathrm{E}[f(\mathbf{d})g(\tilde{r}_e)]$. Function $g(\tilde{r}_e)$ is non-decreasing and, by Assumption (A2), $f(\mathbf{d})$ is also non-decreasing. Moreover, since $(\mathbf{d}, \tilde{r}_e)$ are associated, it follows that

$$\frac{\partial}{\partial q_i}\mathrm{COV}\left(\widetilde{F}(\mathbf{q}), \tilde{r}_e\right) = \mathrm{E}[f(\mathbf{d})g(\tilde{r}_e)] \geq \mathrm{E}f(\mathbf{d})\mathrm{E}g(\tilde{r}_e) = 0,$$

where the last equality follows from $\mathrm{E}g(\tilde{r}_e) = \mathrm{E}[\tilde{r}_e] - r_e = 0$. □

PROOF OF LEMMA 6: We examine the two cases separately.

(a) Immediate from Lemma 5.
(b) If $\mathbf{q} \geq \mathbf{d}_M$, then $\frac{\partial}{\partial q_i}\widetilde{F}(\mathbf{q})$ is independent of the realization of $\mathbf{d}$ (i.e., it is constant in $\mathbf{d}$), hence $\frac{\partial}{\partial q_i}\mathrm{COV}(\widetilde{F}(\mathbf{q}), \tilde{r}_e) = \mathrm{COV}\left(\frac{\partial}{\partial q_i}\widetilde{F}(\mathbf{q}), \tilde{r}_e\right) = 0.$ □

PROOF OF THEOREM 1: It follows from (14) that any non-zero feasible production plan must satisfy the following first order conditions in order to qualify as a maximizer of the shareholders' wealth increase.

$$\frac{\partial}{\partial q_i}NPV^F(\mathbf{q}) - \frac{\partial}{\partial q_i}NPV^O(\mathbf{q}) = 0$$

where

$$\frac{\partial}{\partial q_i}NPV^F(\mathbf{q}) = \frac{\partial}{\partial q_i}\mathrm{E}\widetilde{X}(\mathbf{q}) - \frac{r_e - r_f}{\sigma_e^2}\frac{\partial}{\partial q_i}\mathrm{COV}\left(\widetilde{X}(\mathbf{q}), \tilde{r}_e\right),$$

and

$$\frac{\partial}{\partial q_i}NPV^O(\mathbf{q}) = \eta_0(1 + r_f)\frac{\partial}{\partial q_i}L(\mathbf{q}) \quad (A2)$$

It follows from Lemma 1 that $\frac{\partial}{\partial q_i}\mathscr{L}(\mathbf{q}^N) \geq 0$. Hence, the claim that $\mathbf{q}^W \leq \mathbf{q}^N$ follows from the observation that $\frac{\partial}{\partial q_i}NPV^O(\mathbf{q}) \geq 0$, and if marginal changes in production plan $\mathbf{q}^N$ require no cash, $\frac{\partial}{\partial q_i}\mathscr{L}(\mathbf{q}^N) = 0$, implies $\frac{\partial}{\partial q_i}NPV^O(\mathbf{q}) = 0$ for all i leading to $\mathbf{q}^W = \mathbf{q}^N$. □

PROOF OF THEOREM 2: Claim (1) is immediate from the assumption that $\eta_0 > 0$. To prove claim (2) redefine W^* in (14) as the maximization of $NPV^H(\mathbf{q}) - \eta_0(I1_{\{\mathbf{q}\geq 0\}} + \mathscr{L}^H(\mathbf{q}) + C^H(\mathbf{q}))$. It follows from (18) that the first order conditions for the modified problem become

$$\frac{\partial}{\partial q_i}\mathrm{E}\widetilde{X}(\mathbf{q}) - \frac{r_e - r_f}{\sigma_e^2}\frac{\partial}{\partial q_i}\mathrm{COV}\left(\widetilde{X}(\mathbf{q}), \tilde{r}_e\right) = \eta_0(1 + r_f)\left(\frac{\partial}{\partial q_i}\mathscr{L}^H(\mathbf{q}) + \frac{\partial}{\partial q_i}C^H(\mathbf{q})\right). \quad (A3)$$

Claim (2) follows from comparing the first order conditions in the proof of Theorem 1 with (A3). □

Appendix B: Statistical Assumptions on Product Demand

To characterize the impact of changes in the production plan on risk adjustments, it is useful to establish monotonicity properties of the covariance (positive or negative) of the cash flows to investors with the returns of the market in response to increases in the production plan.

First, we note that assuming demand is positively (negatively) correlated with the returns of the market is not sufficient to guarantee that the cash flows to investors are positively (negatively) correlated with market returns. If demand is positively (negatively) correlated with demand, we can guarantee the cash flows from operation will be positively (negatively) correlated with the market only for the special case where $\widetilde{X}$ is linear and increasing in $\mathbf{d}$. This presents analytical problems because, as in Examples 1 and 2, it is very common for operations management models to produce non-linear cash flows.

To illustrate this point, we present next a simple instance of Example 1 in which, although demand and market returns are positively correlated, the resulting cash flows are negatively correlated with market returns; in fact, their covariance and coefficient of correlation are non-monotonic in the production plan.

EXAMPLE 5 (Correlation of Demand vs. Correlation of Cash Flows). Consider a MTS production system (Example 1) with a single product. The unit sale price, p_1, is \$2, and the cost of production, c_1, is \$1/unit. If the joint probability distribution of demand and market returns is as shown in the last three columns of Table 6, we can readily calculate the coefficient of correlation between demand and market returns as 0.29. Now consider an operating plan that consists of producing 20 units, $q_1 = 20$. The cost of production is \$20, and the cash flows of operations, $\tilde{X}$, is 0, 20, and 20 when demand is 10, 20, and 30 units, respectively. The coefficient of correlation between the cash flow from operations $\tilde{X}(20)$ and the returns of the market can be calculated as -0.22. Thus demand and market returns are positively correlated while the resulting cash flows from operations are negatively correlated with the market returns. We can obtain similarly the cash flows for $q_1 = 10$ and $q_1 = 30$ and their respective covariances and correlation coefficients as shown in the second and fourth column of Table 6.

Similar examples can be constructed in which the demands of two different products are positively correlated, and yet their individual cash flows from operations are negatively correlated (and vice versa). Clearly, this presents serious analytical problems in the characterization of the cash flow beta B, as a function of the production plan $\mathbf{q}$. In this regard, we introduce below the concept of *association* of random variables and *Multivariate Totally Positive of Order 2* MTP_2 distributions as sufficient conditions to induce monotonicity properties on the cash flow beta.

DEFINITION 1 (Associated Random Variables) (Esary et al. 1967). Let $\mathbf{x} = (x_1, \ldots, x_n) \in R^n$ be a vector of random variables. The random variables $(x_1, \ldots, x_n)$ are said to be *associated* if the inequality

$$E[f(\mathbf{x})g(\mathbf{x})] \geq E[f(\mathbf{x})]E[g(\mathbf{x})]$$

is valid for any non-decreasing functions f and g. □

It is clear from Definition 1 that if two random variables are associated, they are positively correlated; however, as Example 5 above illustrates, the converse need not be true. It is not clear from the above definition how one can determine if a set of random variables are associated. In this regard, the class of MTP_2 distributions is of interest for this research because, if the joint probability distribution of a set of variables is MTP_2, then, the random variables are associated.

DEFINITION 2 (MTP_2 Distributions). Let $\mathbf{x} = (x_1, \ldots, x_n) \in R^n$, $\mathbf{y} = (y_1, \ldots, y_n) \in R^n$, and define $\mathbf{x} \vee \mathbf{y} = (\max(x_1, y_1), \ldots, \max(x_n, y_n))$, and $x \wedge y = (\min(x_1, y_1), \ldots, \min(x_n, y_n))$. If ψ is a multivariate density function, we say ψ is MTP_2 if

$$\psi(\mathbf{x} \vee \mathbf{y})\psi(\mathbf{x} \wedge \mathbf{y}) \geq \psi(\mathbf{x})\psi(\mathbf{y}) \qquad (B1)$$

for every $\mathbf{x} \in R^n$ and $\mathbf{y} \in R^n$. □

A sufficient condition to guarantee that a set of random variables are associated is that they follow an MTP_2 probability distribution (Karlin and Rinott 1980). The class of MTP_2 distributions has also been studied in Barlow and Proschan (1975), Karlin and Rinott (1981), and Shaked and Shanthikumar (1994) and, from that research stream, we can determine if a given probability distribution is MTP_2 and hence if its corresponding random variables are associated. For example, if the joint probability distribution of $(\mathbf{d}, \tilde{r}_e)$ is multivariate normal, it is MTP_2 if and only if the off-diagonal elements of $-\Sigma^{-1}$ are all non-negative, where Σ is the covariance matrix of the joint probability distribution. In this case, $(\mathbf{d}, \tilde{r}_e)$ are associated.

Table 6 Joint Probabilities and Correlations

	Demand (ξ)	Operating Income $\tilde{X}(q_1, \xi)$			Joint Probabilities		
		$q_1 = 10$	$q_1 = 20$	$q_1 = 30$	$\tilde{r}_e = -0.1$	$\tilde{r}_e = 0.0$	$\tilde{r}_e = -0.1$
	30	10	20	30	0	0	0.2
	20	10	20	10	0.4	0	0
	10	10	0	−10	0	0.4	0
Covariance	0.16	0	−0.16	0.32			
ρ	0.29	0	−0.22	0.29			

Notes

[1]Shiller (1993, 2003) argues the need to extend the traditional finance analysis to reflect investment opportunities outside the markets.

[2]There are fundamental differences between securities and non-traded factor contracts, and it has been noted (see, e.g., Grossman 1995) that recognizing these differences can have important economic implications. In fact, Coase (1992), in his Nobel address, suggests that bringing in the role of the factors into the theory can shed new light on the institutional structure of production.

[3]Rao and Bharadwaj (2008) have examined the implications of a mixed contracts economy in the context of marketing decisions; unlike this paper, they abstract from investors' risk aversion and the endogenous determination of the firm's optimal policies.

[4]Derivatives and partial derivatives should be interpreted as directional derivatives when the functions are not differentiable, or as finite differences when the functions are discrete. At points where the functions are not differentiable, partial derivatives should be replaced by the sub-differential (the set of sub-gradients) of the function at that point. In this case, the first order necessary conditions for x^* to be a maximum of a function f should be restated to require that 0 is an element of the sub-differential of $-f$ at x^* (Rockafeller 1970). To keep the exposition focused on economic and managerial issues, we abstract in our presentation from these technical issues when they are of no consequence.

[5]There is empirical evidence backing this claim. Longstaff (2004) and Eldor et al. (2006) compare the pricing of risk-free traded securities (T-bills) with the pricing of highly illiquid twin securities (with identical payoffs) and are able to quantify positive "flight-to-liquidity" premiums validating empirically the existence of significant non-tradability premiums even in the absence of risk.

[6]Bajaj et al. (2001) find that there is a discount for privately held ownership. They provide estimates of the non-tradability premiums (termed marketability discounts in their paper). In Table 2, they report premiums varying from 20% to 34% and they suggest that one of the factors that affect the magnitude of the premium is the firm's "business risk." The U.S. Tax Court in *Gross v. Commissioner* 272 F.3d.333 (6th circuit November 19, 2000) accepted the authors' conclusions.

[7]It is important to note that $E^{Q}f_R^N(\omega)$ is well-defined as long as the underlying cash flow distribution $f_R^N(\omega)$ can be replicated with existing (traded) securities. However, one cannot equate $E^{Q}f_R^N(\omega)$ with the value of $f_R^N(\omega)$ unless $f_R^N(\omega)$ is generated by a traded security.

[8]It does not matter whether this cash is obtained from shareholders or creditors. The point is that irrespective of the *source* of these funds the firm must hold them on the balance sheet.

[9]Two strands of economics research (cash-in-advance models and overlapping generations models) admit cash into the theory by imposing an *ad hoc exogenous* requirement that some transactions can only be executed with cash (see, e.g., Kiyotaki and Wright 1989).

[10]We focus on the analysis of production decisions leading to cash flows positively correlated with the market returns as this is the most common case. The case where the cash flows are negatively correlated with market returns mirrors the positively correlated case, and it arises when $(\mathbf{d}, -\tilde{r}_e)$ are associated.

[11]To rationalize the co-existence of positive NPV ownership positions in private businesses simultaneously with investment positions in public firms, we assume that positive NPV ownership opportunities in private firms are limited in size.

[12]The notation $\mathbf{q} \geq 0$ is interpreted here to mean that at least one q_i is strictly positive.

[13]In a competitive economy the mere ability to generate positive NPVs is insufficient for long-term survival. The firm must also generate this NPV most productively, or it is destined to be displaced by competition.

References

Agrawal, V., S. Seshadri. 2000. Impact of uncertainty and risk aversion on price and order quantity in the newsvendor problem. *Manuf. Serv. Oper. Manage.* **2**(4): 410–423.

Anvari, M. 1987. Optimality criteria and risk in inventory models: The case of the newsboy problem. *J. Oper. Res. Soc.* **38**(7): 625–632.

Bajaj, M., D. J. Denis, S. P. Ferris, A. Sarin. 2001. Firm value and marketability discounts. *J. Corp. Law* **27**: 89–115.

Barlow, R. E., R. Proschan. 1975. *Statistical Theory of Reliability and Life Testing: Probability Models.* Holt, Rinehart and Winston, New York.

Berling, P., K. Rosling. 2005. The effects of financial risks on inventory policy. *Manage. Sci.* **51**(12): 1804–1815.

Birge, J. R. 2000. Option methods for incorporating risk into linear capacity planning models. *Manuf. Serv. Oper. Manage.* **2**(1): 19–31.

Birge, J. R., R. Q. Zhang. 1999. Risk-neutral option pricing methods for adjusting constrained cash flows. *Eng. Econ.* **44**(1): 36–49.

Bouakiz, M., M. J. Sobel. 1992. Inventory control with an expected utility criterion. *Oper. Res.* **40**(3): 603–608.

Buzacott, J. A., R. Q. Zhang. 2004. Inventory management with asset-based financing. *Manage. Sci.* **50**(9): 1274–1292.

Caldentey, R., M. Haugh. 2006. Optimal control and hedging of operations in the presence of financial markets. *Math. Oper. Res.* **31**(2): 285–304.

Chamberlain, G., M. Rothschild. 1983. Arbitrage, factor structure, and mean-variance analysis on large asset markets. *Econometrica* **51**(5): 1281–1304.

Coase, R. H. 1992. The institutional structure of production: 1991 Alfred Nobel memorial prize lecture in economic sciences. *Am. Econ. Rev.* **82**(4): 713–719.

Ding, Q., L. Dong, P. Kouvelis. 2007. On the integration of production and financial hedging decisions in global markets. *Oper. Res.* **55**(3): 470–489.

Eeckhoudt, L., C. Gollier, H. Schlesinger. 1995. The Risk-averse (and Prudent) Newsboy. *Manage. Sci.* **41**(5): 786–794.

Eldor, R., S. Hauser, M. Kahn, A. Kamara. 2006. The nontradability premium of derivatives contracts. *J. Bus.* **79**(4): 2067–2097.

Esary, J. D., F. Proschan, D. W. Walkup. 1967. Association of random variables, with applications. *Ann. Math. Stat.* **38**(5): 1466–1474.

Gaur, V., S. Seshadri. 2005. Hedging inventory risk through market instruments. *Manuf. Serv. Oper. Manage.* **7**(2): 103–120.

Grossman, S. J. 1995. Dynamic asset allocation and the informational efficiency of markets. *J. Finance* **50**(3): 773–787.

Gutierrez, G. J., P. Kouvelis. 1995. A robustness approach to international sourcing. *Ann. Oper. Res.* **59**(1): 165–193.

Huchzermeier, A., M. A. Cohen. 1996. Valuing operational flexibility under exchange rate risk. *Oper. Res.* **44**(1): 100–113.

Karlin, S., Y. Rinott. 1980. Classes of orderings of measures and related correlation inequalities. 1. Multivariate totally positive distributions. *J. Mutivar. Anal.* **10**(4): 467–498.

Karlin, S., Y. Rinott. 1981. Total positivity properties of absolute value multinormal variables with applications to confidence interval estimates and related probabilistic inequalities. *Ann. Stat.* **9**(5): 1035–1049.

Kazaz, B., M. Dada, H. Moskowitz. 2005. Global production planning under exchange-rate uncertainty. *Manage. Sci.* **51**(7): 1101–1119.

Kiyotaki, N., R. Wright. 1989. On money as a medium of exchange. *J. Political Econ.* **97**(4): 927–954.

Kouvelis, P., G. J. Gutierrez. 1997. The newsvendor problem in a global market: Optimal centralized and decentralized control policies for a two-market stochastic inventory system. *Manage. Sci.* **43**(5): 571–585.

Lederer, P. J., V. R. Singhal. 1988. Effect of cost structure and demand risk in the justification of new technologies. *J. Manuf. Oper. Manage.* **1**(4): 339–371.

Lee, H. L., C. S. Tang. 1997. Modelling the costs and benefits of delayed product differentiation. *Manage. Sci.* **43**(1): 40–53.

Lee, H. L., C. S. Tang. 1998. Variability reduction through operations reversal. *Manage. Sci.* **44**(2): 162–172.

Longstaff, F. A. 2004. The flight-to-liquidity premium in US treasury bond prices. *J. Bus.* **77**(3): 511–526.

MacMinn, R. D. 1987. Insurance and corporate risk management. *J. Risk Insur.* **54**(4): 658–677.

Rao, R. K. S., N. Bharadwaj. 2008. Marketing initiatives, expected cash flows, and shareholders' wealth. *J. Market.* **72**(1): 16–26.

Rockafeller, R. T. 1970. *Convex Analysis*. Princeton University Press, Princeton, NJ.

Ross, S. A. 1976. The arbitrage theory of capital asset pricing. *J. Econ. Theory* **13**(3): 341–360.

Sethi, S. P. 1997. *Optimal Consumption and Investment with Bankruptcy.* Kluwer Academic Publishers, Norwell, MA.

Shaked, M., J. G. Shanthikumar. 1994. *Stochastic Orders and their Applications*. Academic Press, New York.

Shiller, R. 1993. *Macro Markets: Creating Institutions for Managing Society's Largest Economic Risks*. Oxford University Press, New York.

Shiller, R. 2003. *The New Financial Order*. Princeton University Press, Princeton, NJ.

Singhal, V. R. 1988. Inventories, risk, and vaue of the firm. *J. Manuf. Oper. Manage.* **1**(1): 4–43.

Taksar, M. I. 2000. Optimal risk and dividend distribution control models for an insurance company. *Math. Meth. Oper. Res. (ZOR)* **51**(1): 1–42.

Van Mieghem, J. A. 2003. Commissioned paper: Capacity management, investment, and hedging: review and recent developments. *Manuf. Serv. Oper. Manage.* **5**(4): 269–302.

Van Mieghem, J. A., M. Dada. 1999. Price versus production postponement: Capacity and competition. *Manage. Sci.* 1631–1649.

Xu, X., J. R. Birge. 2004. Joint production and financing decisions: modeling and analysis. Technical Report, Working Paper, Northwestern University.